OCR
Oxford Cambridge and RSA

Thursday 19 June 2025 – Afternoon

A Level Mathematics A

H240/03 Pure Mathematics and Mechanics

Time allowed: 2 hours

188015377

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **16** pages.

ADVICE
- Read each question carefully before you start your answer.

© OCR 2025 [603/1038/8]
DC (DE/FC) 357974/4
OCR is an exempt Charity

Formulae
A Level Mathematics A (H240)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$S_n = \dfrac{a(1-r^n)}{1-r}$

$S_\infty = \dfrac{a}{1-r}$ for $|r| < 1$

Binomial series

$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$(1+x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2 + \ldots + \dfrac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1, n \in \mathbb{R})$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\mathrm{cosec}^2 x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$

Quotient rule $y = \dfrac{u}{v}$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$\mathrm{f}'(x) = \lim\limits_{h \to 0} \dfrac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$

Integration

$\displaystyle\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$

$\displaystyle\int \mathrm{f}'(x)\big(\mathrm{f}(x)\big)^n\,\mathrm{d}x = \frac{1}{n+1}\big(\mathrm{f}(x)\big)^{n+1} + c$

Integration by parts $\displaystyle\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

© OCR 2025

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k + \frac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\displaystyle\int_a^b y\,\mathrm{d}x \approx \tfrac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$ **or** $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Standard deviation

$\sqrt{\dfrac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\dfrac{\Sigma x^2}{n} - \bar{x}^2}$ **or** $\sqrt{\dfrac{\Sigma f(x-\bar{x})^2}{\Sigma f}} = \sqrt{\dfrac{\Sigma f x^2}{\Sigma f} - \bar{x}^2}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = x) = \dbinom{n}{x} p^x (1-p)^{n-x}$, mean of X is np, variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\overline{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\overline{X} - \mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that $\mathrm{P}(Z \leqslant z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line	Motion in two dimensions
$v = u + at$	$\mathbf{v} = \mathbf{u} + \mathbf{a}t$
$s = ut + \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$
$s = \frac{1}{2}(u + v)t$	$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$
$v^2 = u^2 + 2as$	
$s = vt - \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

Section A
Pure Mathematics

1 **(a)** Express $3x^2 - 12x + 17$ in the form $a(x-b)^2 + c$ where a, b and c are constants. **[3]**

(b) State the coordinates of the minimum point of the curve $y = 3x^2 - 12x + 17$. **[1]**

(c) State the equation of the normal to the curve $y = 3x^2 - 12x + 17$ at its minimum point. **[1]**

2 **(a)** The curve $y = \frac{1}{x^3}$ is translated by 4 units in the positive x-direction.

Write down the equation of the curve after it has been translated. **[2]**

(b) The curve $y = 5x^2$ is stretched parallel to the y-axis with scale factor 4.

The point on the curve $y = 5x^2$ with x-coordinate 3 is transformed to the point P.

Write down the coordinates of P. **[2]**

© OCR 2025

3 **(a)** Sketch the graph of $y = |2x - 5|$ on the grid provided in the Printed Answer Booklet. **[2]**

(b) Solve the inequality $|2x - 5| < 1$. **[2]**

(c) **In this question you must show detailed reasoning.**

Hence find **all** the possible integer values N that satisfy the inequality

$|2e^{0.1N} - 5| < 1$. **[3]**

4 Functions f and g are defined for all real values of x by

$f(x) = \frac{x-k}{2}$ and $g(x) = x^2 + kx + 5$, where k is a constant.

You are given that the equation $f^{-1}g(x) = 4 - 2kx$ has real distinct roots.

(a) Show that k satisfies the inequality $2k^2 - k - 6 > 0$. **[5]**

(b) **In this question you must show detailed reasoning.**

Hence find the set of values of k. Give your answer in set notation. **[3]**

5 **(a)** Use the substitution $u = \cos x$ to show that

$$\int \frac{1+\cos x}{\sin x}\,\mathrm{d}x = \ln(1-\cos x) + c.$$ **[4]**

Fig. 1

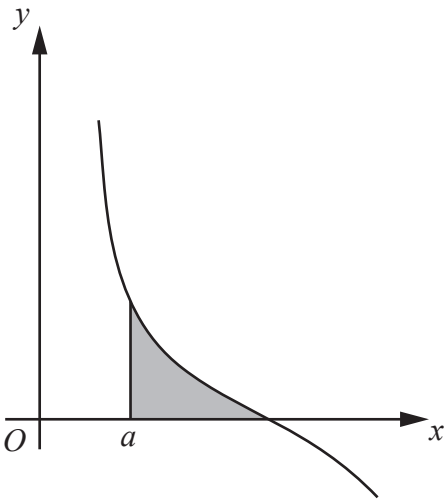


Fig. 1 shows part of the curve $y = \dfrac{1+\cos x}{\sin x}$.

The shaded region is bounded by the curve, the x-axis, and the line $x = a$.

You are given that the area of the shaded region is a square units.

(b) Show that the value of a satisfies the equation $a - \cos^{-1}(1-2\mathrm{e}^{-a}) = 0$. **[4]**

(c) Show by calculation that the value of a lies between 1.1 and 1.2. **[2]**

(d) Use the iterative formula

$$a_{n+1} = \cos^{-1}(1-2\mathrm{e}^{-a_n}),$$

with starting value $a_1 = 1.2$, to find the value of a correct to **2** decimal places. Show the result of each step of the iterative process. **[2]**

© OCR 2025

Fig. 2

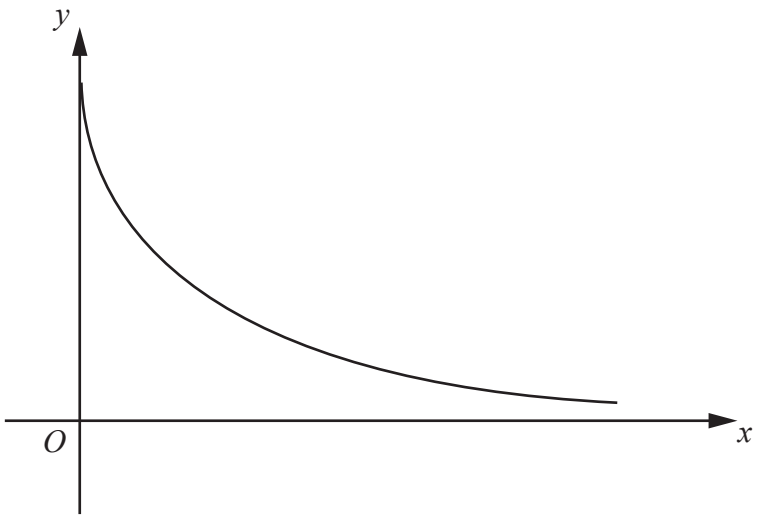


Fig. 2 shows the curve $y = \cos^{-1}(1 - 2e^{-x})$.

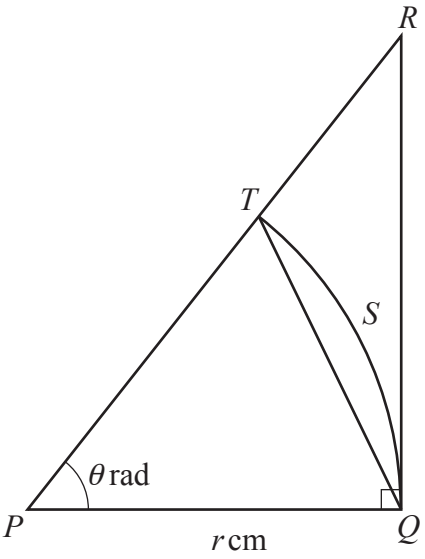
(e) Explain why the gradient of the curve $y = \cos^{-1}(1 - 2e^{-x})$ at the point where $x = a$, where a is the value found in part **(d)**, lies in the interval $(-1, 0)$. **[2]**

6 The compound angle formulae for $\sin(A+B)$ and $\sin(A-B)$ are

$\sin(A+B) = \sin A\cos B + \cos A\sin B$ and
$\sin(A-B) = \sin A\cos B - \cos A\sin B$.

(a) By letting $C = A + B$ and $D = A - B$, show that

$$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right).$$ **[1]**



The diagram shows a right-angled triangle PQR with $PQ = r$ cm. The angle QPR is θ radians. The diagram also shows the sector $PQST$ of a circle with centre P and radius r cm. The line segment QT is a chord of the sector $PQST$.

(b) By considering the areas of triangle PQT, sector $PQST$ and triangle PQR, show that

$$1 < \frac{\theta}{\sin\theta} < \frac{1}{\cos\theta}.$$ **[4]**

(c) (i) Hence write down a similar inequality interval for the expression $\frac{\sin\theta}{\theta}$. **[1]**

(ii) Hence state $\lim_{\theta\to 0}\frac{\sin\theta}{\theta}$. **[1]**

(d) Using parts **(a)** and **(c)(ii)**, show from first principles that the derivative of $\sin x$ is $\cos x$, where x is measured in radians. **[4]**

A student attempts to use the result regarding the derivative of $\sin x$ to find the derivative of $\cos x$. The student's attempt is shown below.

Let	$y = \cos x$, where x is measured in radians.
	$y = \sin\left(\frac{\pi}{2} - x\right)$
so	$\frac{\mathrm{d}y}{\mathrm{d}x} = \cos\left(\frac{\pi}{2} - x\right)$
but	$\sin x \equiv \cos\left(\frac{\pi}{2} - x\right)$
therefore	$\frac{\mathrm{d}y}{\mathrm{d}x} = \sin x$.

(e) Identify the error made by the student. **[1]**

BLANK PAGE

© OCR 2025

Section B
Mechanics

7 A particle P of mass 3 kg is moving on a smooth horizontal surface under the action of two constant horizontal forces $(5\mathbf{i}+3\mathbf{j})$ N and $(a\mathbf{i}+3b\mathbf{j})$ N. The acceleration of P is $(2\mathbf{i}-3\mathbf{j})\,\mathrm{m\,s^{-2}}$.

(a) Find the value of a and the value of b. **[3]**

At time $t = 0$ seconds the velocity of P is $\mathbf{u}\,\mathrm{m\,s^{-1}}$ and at time $t = 5$ seconds the velocity of P is $(7\mathbf{i}-6\mathbf{j})\,\mathrm{m\,s^{-1}}$.

(b) Find, in terms of **i** and **j**, an expression for **u**. **[2]**

8



Two particles A and B of masses 0.6 kg and 0.8 kg respectively, are attached to the ends of a light inextensible string. Particle A is held at rest at a fixed point and B hangs vertically below A.

Particle A is now released. As the particles fall the air resistance acting on A has a constant magnitude of R N and the air resistance acting on B has a constant magnitude of 0.3 N. The string remains taut throughout the subsequent motion (see diagram).

The downward acceleration of each of the particles is $9.2\,\mathrm{m\,s^{-2}}$ and the tension in the string is T N.

(a) By applying Newton's second law to B, find the value of T. **[2]**

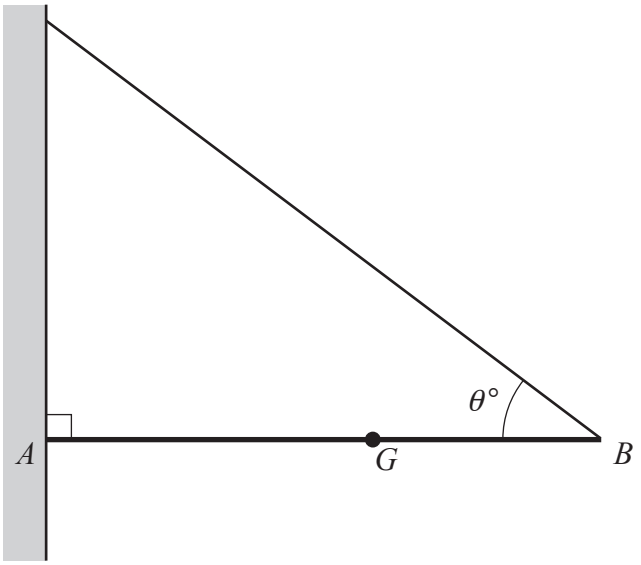
(b) Hence, or otherwise, find the value of R. **[2]**

9 **In this question you must show detailed reasoning.**

A particle moves in a straight line. Its velocity t seconds after leaving a fixed point on the line is $v\,\text{m s}^{-1}$ where $v = 2 + 3t + 0.6t^2$.

Find the distance travelled by the particle from time $t = 0$ until the instant when its acceleration is $15\,\text{m s}^{-2}$. **[6]**

10



The diagram shows a **non-uniform** rod AB of mass 4 kg and length 2 m. The end A of the rod rests against a rough vertical wall. The rod is held in a horizontal position, perpendicular to the wall, by a light inextensible string attached to the rod at B. The other end of the string is attached to the wall at a point vertically above A. The string is inclined at an angle of $\theta°$ to the horizontal, where $\sin\theta = \frac{4}{5}$.

The rod rests in equilibrium in a vertical plane perpendicular to the wall. The rod's weight acts at the point G on AB.

You are given that the magnitude of the moment of the rod's weight about A is 47.04 N m.

(a) Show that the distance AG is 1.2 m. **[1]**

(b) Find the tension in the string. **[2]**

(c) Determine the magnitude of the contact force exerted on the rod by the wall at A. **[5]**

© OCR 2025

11 In this question you should take the acceleration due to gravity to be $10\,\text{m s}^{-2}$.

The unit vectors **i** and **j** are in a vertical plane with **i** horizontal and **j** vertically upwards.

A small ball P is projected from a point O on horizontal ground into the air. When P is in the air, it is to be modelled as a particle moving under the influence of gravity only.

At time $t = 2$ seconds, P has velocity $(2\mathbf{i} - 8\mathbf{j})\,\text{m s}^{-1}$.
The position vector of a point on the trajectory of P is $(x\mathbf{i} + y\mathbf{j})$m relative to O.

(a) Show that $y = 6x - 1.25x^2$. **[5]**

(b) Determine the maximum height of P above the ground during its motion. **[3]**

(c) Determine the following as P passes through the point on its trajectory where $x = 2.5$.

- the speed of P
- the direction of motion of P **[5]**

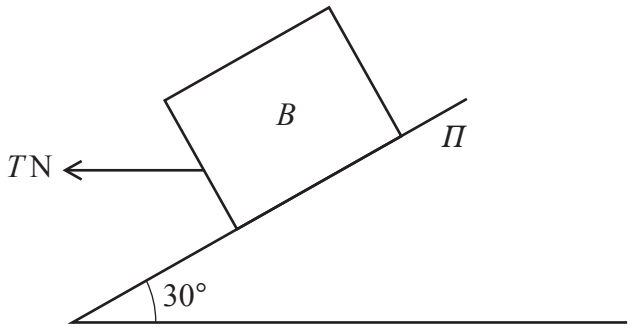
In reality P does not move only under the influence of gravity but is also subject to air resistance.

(d) Explain how this would affect your answer to part **(b)**. **[1]**

Turn over for question 12

12

Fig. 1



A rectangular block B of mass 10 kg lies at rest in limiting equilibrium on a rough plane Π inclined at 30° to the horizontal. A horizontal force of magnitude T N, acting above a line of greatest slope, is applied to B (see **Fig. 1**).

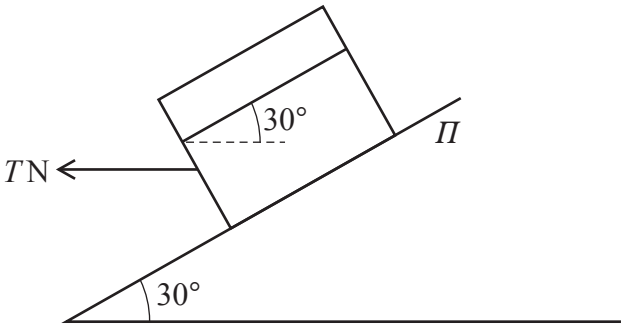
The coefficient of friction between B and the plane is 0.8.

(a) Show that the value of T is 14.9, correct to **3** significant figures. **[6]**

© OCR 2025

For the remainder of the question, you should you use this value of *T*.

Fig. 2



Block *B* is now cut at an angle of 30° to the horizontal into two smaller blocks. The upper block has a mass of 4 kg, and the lower block has a mass of 6 kg. The two blocks are held at rest with the lower block on Π. The horizontal force of magnitude T N is now applied to the lower block (see **Fig. 2**).

The two blocks are released from rest and in the subsequent motion the upper block starts to move with acceleration $3.5\,\text{m}\,\text{s}^{-2}$.

(b) Determine the coefficient of friction between the two blocks. **[4]**

(c) Show that the lower block does **not** move. **[3]**

END OF QUESTION PAPER

OCR
Oxford Cambridge and RSA

Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact The OCR Copyright Team, The Triangle Building, Shaftesbury Road, Cambridge CB2 8EA.

OCR is part of Cambridge University Press & Assessment, which is itself a department of the University of Cambridge.

© OCR 2025